Gravitational Waves from Axions

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Outline

- Prospects of gravitational wave detection

- Axions and axion strings

- String dynamics and generation of gravitational waves
  - Bounds on axion mass and decay constant

- Conclusion and Outlook
Gravitational Waves

![Gravitational Waves Image]

![Graph showing strain vs. time]
The diagram illustrates the search for modified gravity models (BSM) in the context of gravitational wave (GW) observations. The x-axis represents the frequency $f$ in Hz, while the y-axis shows the GW strain $\frac{d\Omega_{gw} h^2}{d \log f}$. The graph is divided into different regions, each labeled with various interferometer projects such as PTA, SKA, USA, IAO, and others.

Key features include:
- **Pulsar Timing Arrays**: Used for detecting low-frequency GWs.
- **Space-based Interferometers**: Complement ground-based observations by reaching higher frequencies.
- **Ground-based Interferometers**: Essential for lower frequency observations.

The graph highlights the importance of these technologies in advancing our understanding of the universe, particularly in probing the validity of general relativity and searching for BSMs.
Axion \( \equiv a \)

- Goldstone boson of a (new) spontaneously broken U(1) symmetry (at the scale \( f_a \))

  Simplest realization:
  \[
  \phi = \frac{1}{\sqrt{2}} (r + f_a) e^{i \frac{\phi}{f_a}}
  \]
  \[
  V_\phi = \frac{m_a^2}{2f_a^2} \left( |\phi|^2 - \frac{f_a^2}{2} \right)^2
  \]

- U(1) explicitly broken by the axion potential \( V(a) \)
  \[
  \rightarrow V(a) \text{ invariant under } a \rightarrow a + 2\pi f_a
  \]
  \[
  \rightarrow \text{axion mass } m_a
  \]

**Example**

QCD axion:

\[
\mathcal{L} \supset \frac{a}{f_a} G_{\mu\nu} \tilde{G}^{\mu\nu}
\]

\[
\rightarrow m_a \simeq m_\pi f_\pi / f_a
\]

Strong CP:

\[
\mathcal{L}_{SM} \supset \theta_{\text{QCD}} G_{\mu\nu} \tilde{G}^{\mu\nu}
\]

\[
\lesssim 10^{-11}
\]

\[
\rightarrow \theta_{\text{QCD}} \frac{(a)}{f_a} \rightarrow 0
\]
Cosmological Initial Conditions

Pre-inflationary

\[ T_R \lesssim f_a \text{ and } H_I \lesssim f_a \]

\[ \theta \equiv \frac{a}{f_a} \in [-\pi, \pi] \]

Post-inflationary

\[ T_R \gtrsim f_a \text{ or } H_I \gtrsim f_a \]
Origin of the inhomogeneities in the post-inflationary scenario

- Finite temperature corrections
  \[ T_{RH} \lesssim 10^{15} \text{ GeV} \]

- Quantum inflationary fluctuations:
  \[ \langle \sigma_a^2 \rangle = \left( \frac{H_I}{2\pi} \right)^2 \rightarrow H_I \gtrsim \frac{f_a}{2\pi} \]

\[ \frac{H_I}{2\pi} \lesssim 10^{13} \text{ GeV} \]

- Direct coupling of \( \phi \) to the inflation \( \varphi \)?
  \[ V_\phi \supset g\varphi^2|\phi|^2 \quad \text{during inflation} \]
  \[ \rightarrow g\langle \varphi \rangle^2|\phi|^2 \]
  effective mass for \( \phi \)

\[ g\langle \varphi \rangle^2 \gtrsim m_r^2 \]
\[ \mathcal{L} = |\partial_\mu \phi|^2 - V_\phi \]
\[ V_\phi = \frac{m_r^2}{2f_a^2} \left( |\phi|^2 - \frac{f_a^2}{2} \right)^2 \]

Trivial inhomogeneous solutions:

\[
\begin{cases} 
  r = 0 \text{ (radial mode on its VEV)} \\
  \partial_\mu \partial^\mu a = 0
\end{cases}
\]

Axion waves:
\[ \phi = \frac{f_a}{\sqrt{2}} e^{ik(t-x)} \]
Strings

Axion waves:
\[ \phi = \frac{f_a}{\sqrt{2}} e^{i k(t-x)} \]

\[ \phi = \frac{f_a}{\sqrt{2}} g(|x|) e^{i \theta} \]

\[ \begin{cases} 
  g(0) = 0 \\
  g(\infty) = 1
\end{cases} \]

String tension:
\[ \mu = \frac{E}{L} \sim \pi f_a^2 \log \frac{d}{m_r^{-1}} \sim \pi f_a^2 \log \frac{m_r}{H} \]

grows logarithmically in time

String core:
\[ m_r^{-1} \sim f_a^{-1} \]

Nonlinear dynamics:
- Analytical approach ☹

Large ratio of scales:
- Numerical approach 😞
The Scaling Regime

causal patch $\propto 1/H = 2\ell$
The Scaling Regime

causal patch $\propto 1/H = 2t$

free strings: $\rho_{\text{free}} \propto \frac{1}{R^2} \propto \frac{1}{t}$
The Scaling Regime

causal patch $\propto 1/H = 2t$

rate of energy loss: $\Gamma \equiv \frac{d}{dt} \left[ \rho^\text{free} - \rho^\text{scal} \right] \propto \frac{\xi \mu}{t^3}$

$\propto \frac{1}{R^2} \propto \frac{1}{t} \quad \frac{\xi \mu}{t^2}$ number of strings per Hubble patch

$H^{-1}$ for $\xi = 1, 2, < 1$
\( T,H \geq f_a \rightarrow \text{BBN [ultralight]} \quad H \sim m_a (T \sim \Lambda_{\text{QCD}}) \rightarrow \text{BBN [QCD axion], CMB} \quad \text{today} \)

\[ \Gamma_a = \Gamma \]

\[ |\log(m_r/H)| \sim 1 \div 15 \quad \sim 70 \div 100 \]

strings form and annihilate

relic axions and gravitational waves

strings \quad \text{and} \quad \text{domain walls}
Scaling Violation

\[ \xi = c_1 \log + c_0 + \frac{c-1}{\log} + \frac{c-2}{\log^2} \]

\[ \Gamma_a = \frac{\xi \mu}{t^3} \propto \frac{f_a^2 \log^2}{t^3} \]
Gravitational Waves

\[ T, H \geq f_a \quad H \sim m_a (T \sim \Lambda_{\text{QCD}}) \quad \text{today} \]

\[ \log(m_r/H) \sim 1 \div 15 \]
\[ \Gamma_a = \Gamma \]
\[ \Gamma_g = ? \sim 70 \div 100 \]

GWs from the scaling regime

1) GW energy emission rate: \( \Gamma_g(t) \)

2) momentum distribution: \( \frac{\partial \Gamma_g}{\partial k} \)

Nambu–Goto EFT: \( \Gamma_g \leftrightarrow \Gamma_a \)
String Effective Theory

Degrees of freedom:

- $a \leftrightarrow A_{\mu\nu}$
- $X^\mu(\tau, \sigma)$

\[
\partial A \sim F^{\mu\nu\rho} = \epsilon^{\mu\nu\rho\sigma} \partial_\sigma a
\]

\[
S_{\phi}[\phi] \quad \downarrow \quad S_{\text{EFT}}[X, A] = -\mu \int d\tau d\sigma \sqrt{-\gamma} - \frac{1}{6} \int d^4x (\partial A)^2 + 2\pi f_a \int d\tau d\sigma \partial_\tau X^\mu \partial_\sigma X^\nu A_{\mu\nu}
\]

- Nambu–Goto action $\gamma_{ab} = \partial_a X^\mu \partial_b X^\mu$
- Axion kinetic term
- Axion-string interaction (Kalb–Ramond action)

1) GW energy emission rate: $\Gamma_a(t)$
2) momentum distribution: $\frac{\partial}{\partial \phi}$
Validity of the EFT breakdown
Gravitational Wave Emission

EoM for $A^{\mu\nu}$:
$$\Box_x A^{\mu\nu} = 2\pi f_a \int d\sigma \dot{X}^{[\mu} X^{\nu]} \delta^3(x - \bar{x})$$

Einstein Eq:
$$\Box_x h^{\mu\nu} = 16\pi G \left( T_s^{\mu\nu} - \frac{1}{2} \eta^{\mu\nu} T_s^{\lambda\lambda} \right)$$
$$T_s^{\mu\nu} = \int d\sigma \left( \dot{X}^\mu \dot{X}^\nu - X'^\mu X'^\nu \right) \delta^3(x - \bar{x})$$

$$\frac{dE_a}{dt} = r_a[X] f_a^2$$
$$\frac{dE_g}{dt} = r_g[X] G \mu^2$$

dimensionless functionals of the shape of the string trajectory $X^\mu$

$$\frac{\Gamma_g}{\Gamma_a} = \frac{r_g[X]}{r_a[X]} \frac{G \mu^2}{f_a^2}$$
$$\equiv r = \text{const}$$

$$\Gamma_g = r \frac{G \mu^2}{f_a^2} \Gamma_a$$
$$\propto \frac{\log^4 t}{t^3}$$
Comparison with the Field Theory Evolution

1) GW energy emission rate: $\Gamma_g(t)$
2) momentum distribution: $\frac{dt}{dt}$

Total GW energy:

$$\rho_g = \int dt' \left( \frac{R'}{R} \right)^4 \Gamma_g' \propto \frac{\log^2}{t^2}$$
The Gravitational Wave Spectrum

\[ \frac{d\Gamma_g}{dk} \]

1) GW energy emission rate: \( \Gamma_g(t) \)
2) momentum distribution: \( \frac{\partial n}{\partial k} \)
The Gravitational Wave Spectrum

\[ \frac{d\Gamma_g}{dk} \]

\[ \frac{\partial \rho_g}{\partial \log k} \equiv \int dt' \frac{d\Gamma_g'}{d\log k} \left( \frac{R'}{R} \right)^4 \]

\[ \approx 8\pi^3 r G \int \frac{m_r}{H} \left( \frac{x_0 H}{k} \right)^2 \log^4 \left( \frac{m_r}{H} \left( \frac{x_0 H}{k} \right)^2 \right) \]

1) GW energy emission rate: \( \Gamma_g(t) \)
2) momentum distribution: \( \frac{\partial \rho_g}{\partial \log k} \)

- approximately scale invariant
- \( \log^4 \) enhancement
\[
\frac{d\Omega_{gw} h^2}{d \log f} \simeq 10^{-15} \left( \frac{r}{0.26} \right) \left( \frac{f_a}{10^{14} \text{GeV}} \right)^4 \left( \frac{10}{g_f} \right)^{\frac{1}{2}} \left\{ 1 + 0.12 \log \left[ \left( \frac{m_r}{10^{14} \text{GeV}} \right) \left( \frac{10^{-5} \text{Hz}}{f} \right)^2 \right] \right\}^4
\]

\[
\begin{align*}
f_a & \lesssim 10^{15} \text{ GeV} \\
m_a & \lesssim 10^{-18} \text{ eV}
\end{align*}
\]
Bounds on the Post-Inflationary Scenario

- isocurvature perturbations
- dark radiation
- dark matter

\[ \Omega^{\text{st}}_a \approx 0.1 \left( \frac{\xi \, \log_{10}}{3 \times 10^8} \right) \left( \frac{f_a}{10^{14} \text{ GeV}} \right)^2 \left( \frac{m_a}{10^{-18} \text{ eV}} \right)^{\frac{1}{3}} \]
Conclusions

- Axions are motivated BSM candidates
  \[ \rightarrow \text{in the post-inflationary scenario, the cosmological evolution is governed by cosmic strings} \]

- The scaling regime produces an approximately scale invariant GW spectrum
  \[ \rightarrow \Gamma_g \propto \log^4 \] (from the increase in \( \mu \propto \log \) and \( \xi \propto \log \)) leads to logarithmic violations of scale invariance
  \[ \rightarrow \text{enhances the spectrum at low frequencies} \]

- The spectrum is visible by multiple experiments for \( f_a > 10^{14} \text{ GeV} \)
  \[ \rightarrow \text{best prospects in PTAs and LISA} \]

- Constraints on the post-inflationary scenario
  \[ \rightarrow f_a \lesssim 10^{15} \text{ GeV} \text{ and } m_a = 10^{-28} \div 10^{-18} \text{ eV is viable} \]

Outlook

- Local strings?
- (Initial conditions for the subsequent evolution?)